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Power-law relaxation in human violent conflicts

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Abstract. We study relaxation patterns of violent conflicts after bursts of activity. Data were obtained from available catalogs on the conflicts in Iraq, Afghanistan and Northern Ireland. We find several examples in each catalog for which the observed relaxation curves can be well described by an asymptotic power-law decay (the analog of the Omori's law in geophysics). The power-law exponents are robust, nearly independent of the conflict. We also discuss the exogenous or endogenous nature of the shocks. Our results suggest that violent conflicts share with earthquakes and other natural and social phenomena a common feature in the dynamics of aftershocks.

1 Introduction

Non-exponential relaxation to a typical state has been observed in several physical systems [1–5] including earthquakes [6–8] and fracturing phenomena [9–11]. A general law governing relaxation in earthquake-like phenomena is the Omori's law [6], which states that the number of events per unit time decreases as a power-law since the sudden rise of activity triggered by a shock. Omori-like decay has also been observed in a wide range of social systems, including download rates [12], internet traffic [13], financial markets [14–17], book sales [18–20], views of videos [21], spamming [22] and words in online blogs [23]. Despite strong evidences pointing to the occurrence of Omori-like decay in the social context, there are few works devoted to a quantitative characterization of power-law relaxation in violent activities (for counterexamples see Refs. [24,25]).

Here we study relaxation patterns in human violent conflicts. The database was obtained from records on the conflicts in Iraq, Afghanistan and Northern Ireland. We find several examples of bursts of activity (shocks) for which the observed relaxation curves can be well described by an asymptotic power-law decay (like the Omori's law). We verify that the power-law exponents are robust, nearly independent of the conflict. We also discuss the nature of the bursts (endogenous or exogenous) by comparing the relaxations curves with the predictions of a general model of self-excited processes proposed some years ago. Finally, we discuss the similarities between our findings and those obtained in natural and social relaxation phenomena.

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2 Data and methods

Iraq data refers to a period of 803 days within 2003–2005 and were acquired from the supplementary material of reference [26]. Afghanistan data cover a period of 1056 days, from January 2008 to December 2010 [27]. Northern Ireland data refers to a catalog covering 11 839 days, from July 1969 to December 2001 [28]. Iraq and Afghanistan databases consist of violent events of size s (the estimated number of deaths) and the day t that the event occurred or that the event started in the case of multi-day attacks. Northern Ireland database provides information on single conflict-related deaths (only events of size $s = 1$) including the day t that it occurred. We have obtained events of size $s \geq 1$ by summing up all the single deaths that occurred on the same day. For this reason, only one event of size $s \geq 1$ was associated with a given violent day t for Northern Ireland data while for Iraq and Afghanistan data one or more events can correspond to the same day. We choose the time t in such way that $t = 1$ corresponds to the first day in each database.

For a given catalog, we selected events of size $s > s_{\min}$ occurring at times $t = T$. Each selected event, which has no larger event in the time interval $[T - \Delta t^*, T + \Delta t^*]$, where Δt^* is an integer (an estimate of the time of relaxation Δt), was called mainshock. Events occurred after a mainshock were called aftershocks ($t - T > 0$) and events occurred before the mainshock, foreshocks ($t - T < 0$). We also obtained the aftershock rate r versus the time to the mainshock $t - T$ for each selected event. The rate r was calculated by dividing the time interval $[T, T + \Delta t]$ in intervals (or bins) of size τ, counting the total number of events within each bin and dividing by τ. Here, τ is an integer representing a time interval in days.

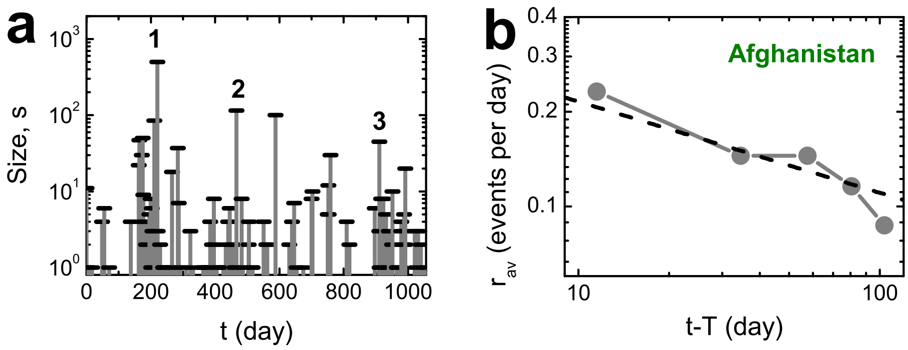

Fig. 1. Selected mainshocks (triggering events) and the correspondent average aftershock rates for the Afghanistan catalog. (a) Event size s, measured in conflict-related deaths, versus the time t shown for all days in the catalog. The three selected triggering events are indicated as events 1, 2 and 3 at times $T = 220$, $T = 466$ and $T = 911$ respectively. (b) Average aftershock rates, r_{av}, obtained by averaging the individual rates r_1, r_2 and r_3, versus the time to the mainshock $t - T$. The dashed line is a power-law decay given by equation (1), with $p = 0.29$ and $a = 0.42$ (see Tab. 2).

We investigated the time evolution $r(t-T)$ indirectly by analysing the cumulative rate defined as $n(t-T) = \int_T^t r(t'-T)dt'$. The analysis of the cumulative rate was chosen since n is less noisy and typically more suitable for comparison with the empirical data. Suppose that we have a power-law decay (like the Omori's law in geophysics) given by

$$r \sim a\,(t-T)^{-p}, \qquad (1)$$

where a is a positive constant and p is the power-law exponent (Omori exponent) for the aftershock rate. The corresponding cumulative rate can be written as

$$n \sim b\,(t-T)^{1-p}, \qquad (2)$$

where $b = a/(1-p)$. If r is constant in time, like a stochastic process with no memory, we have $p \simeq 0$ and the cumulative rate grows linearly with time. Similar definitions hold for the foreshock rate $r'(|t-T|)$ and the cumulative foreshock rate $n'(|t-T|)$, with $t - T < 0$.

The time of relaxation Δt is a priori unknown for a given mainshock. Large values of Δt provide us more data but we need to distinguish between the relaxation process, where the activity is a decaying function of time, and any possible subsequent background noise. For this reason we choose Δt according to the following procedure. We defined $\Delta t = k\tau$, where k is an integer corresponding to the total number of bins of size τ used. For fixed k, we range τ from a lower bound $\tau_{\min}$ to an upper bound τ_{max} obtaining a set of Δt values. For each Δt we calculated an empirical curve $r(t-T)$, excluding the curves for which some bin has no count (if, for all Δt values, some bin has no count we excluded this mainshock from the analysis). From the remaining curves, we obtained a set of $n(t-T)$ curves and performed least squares fits to equation (2). We then choose the Δt value that led to the best fit (maximum coefficient of determination R^2) of $n(t-T)$ to equation (2).

By fitting the best $n(t-T)$ curve to equation (2), we obtained the Omori exponent p and the standard error se (related to p) for each mainshock in a given catalog. We selected the mainshock if p is positive ($p \pm se > 0$).

Table 1. Selected mainshocks (triggering events) per catalog. We list the size s of the event (measured in conflict-related deaths) and the day T in which the event occurred (counted from the first day of the catalog). In the selection procedure, values of $\{s_{\min}, \Delta t^*, k, \tau_{\min}, \tau_{\max}\}$ are as follows: $\{5, 120, 5, 15, 25\}$ for the Afghanistan, $\{5, 50, 6, 4, 8\}$ for the Iraq and $\{5, 50, 5, 6, 8\}$ for the N. Ireland catalog.

Event	Catalog	T	s
1	Afghanistan	220	500
2	Afghanistan	466	115
3	Afghanistan	911	45
4	Iraq	168	31
5	Iraq	313	616
6	Iraq	442	84
7	Iraq	530	670
8	N. Ireland	756	14
9	N. Ireland	873	15
10	N. Ireland	930	15
11	N. Ireland	1255	8
12	N. Ireland	1403	8
13	N. Ireland	1956	22
14	N. Ireland	2091	8
15	N. Ireland	3696	23
16	N. Ireland	4327	5
17	N. Ireland	6691	12

We repeated this procedure for each mainshock in a given catalogue. From a total of 31 mainshocks with enough aftershocks to extract the power-law exponents, 17 were selected (see Tab. 1).

Instead of analysing the relaxation process after each triggering event separately, we performed averages of $r(t-T)$ over all selected mainshocks in a given catalog to obtain the mean activity rate $r_{av}(t-T)$ and $n_{av}(t-T) = \int_T^t r_{av}(t'-T)dt'$. The optimal Δt was obtained by using the same procedure to analyze individual curves. Figure 1 shows the three selected mainshocks for the Afghanistan catalog (see Tab. 1) and the correspondent average relaxation curve r_{av} versus $t - T$ in comparison with equation (1).

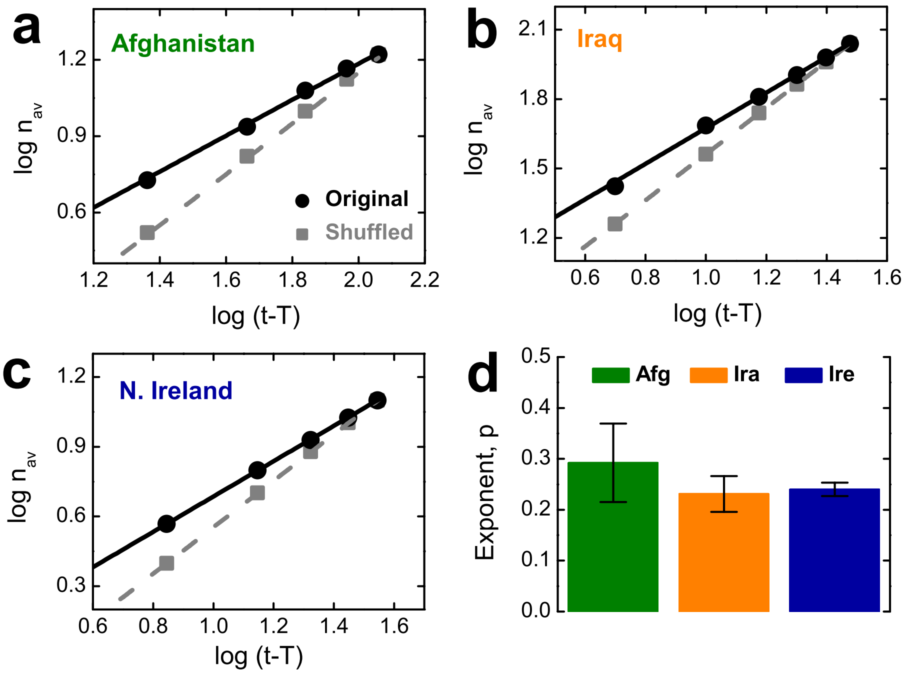

Fig. 2. Quantifying the Omori-like relaxation. Logarithmic of the averaged accumulated aftershock rate, $\log n_{av}$ (black circles), as a function of the logarithmic of the time to the mainshock, $\log(t-T)$. The data are shown in comparison with least squares fits to equation (2) (solid lines) for (a) Afghanistan; (b) Iraq; and (c) N. Ireland catalogs. The fitted power-law exponents p are shown in (d) with their correspondent 95% confidence intervals (see also Tab. 2). We also show the averaged cumulative rate obtained from shuffled data (gray squares) in comparison with least squares fits to equation (2) (dashed lines). For all cases, the averaged control curves (shuffled data) display exponents $\simeq$0, as expected for a stochastic processes with no memory.

3 Results

Figure 2 shows $\log n_{av}$ versus $\log(t-T)$, obtained from the mainshocks described in Table 1, for all the empirical catalogs. The empirical curves are shown in comparison with least square fits to equation (2). The fitted power-law exponents p, with their confidence intervals, are shown in Figure 2d. These findings point to Omori exponents of the order of 0.2–0.4 for all catalogs analyzed. There are good agreement between equation (2) and the empirical data, with $R^2 \geq 0.9997$ (see Tab. 2). In addition, we test if the fit residuals are normally distributed. By using a Pearson χ^2 goodness-of-fit test, we verified that the null hypothesis that the residuals are distributed according to the normal distribution is not rejected at the 0.05 confidence level (p-values are shown in Tab. 2).

Next we investigated the role of random fluctuations in obtaining the power-law exponents p. We generated control curves by shuffling in time all the aftershocks following a given mainshock and performing a parallel analysis to obtain shuffled versions of $n_{av}(t-T)$. These shuffled versions of $n_{av}(t-T)$, averaged over several realizations, are shown in Figure 2 in comparison with the correspondent original curves. Least squares fits of the averaged control curves to equation (2) give Omori exponents $\simeq$0, as expected for a stochastic process with no memory. Fits performed individually, for each shuffled realization, led to a set of exponents p_{sh} which fluctuate around zero (see Tab. 2). A comparison between p and p_{sh} suggests that the observed Omori decay can not be explained just assuming random fluctuations.

We also compared the performance of the proposed Omori-like decay to describe the empirical data with the performance of an exponential decay given by $r_{av} \sim c\exp[-t/t_0]$. We fitted the empirical $n_{av}(t-T)$ to the correspondent exponential curve given by $n_{av} \sim c'(1-\exp[-(t-T)/t_0])$, with $c' = ct_0$, in the same range of times used in the previous analysis. By using Akaike Information Criterion (AIC) corrected for finite samples, typically we obtained negative values for δAIC pointing to the Omori-like decay as the best model (see Tab. 2). For N. Ireland and Iraq data, the evidence for the power-law relaxation is more strong in comparison with Afghanistan data.

We tested the robustness of the measured Omori exponent with the cutoff $s_{\min}$. Figure 3 shows p versus $s_{\min}$, with their correspondent confidence intervals, for all the empirical catalogs. It is apparent that the power-law exponents stay approximately constant against $s_{\min}$, indicating that the observed Omori-like relaxation is robust. Naturally, larger values of $s_{\min}$ lead to a smaller number of selected triggering events. This tends to increase the error bars.

We performed a parallel analysis for the foreshocks obtaining the cumulative rate n'_{av} versus $|t-T|$ (with $t-T<0$) for the same selected triggered events shown in Table 1. According to our results from the analysis of the foreshocks, shown in Table 2, only the Afghanistan data presented a power-law exponent distinguishable from

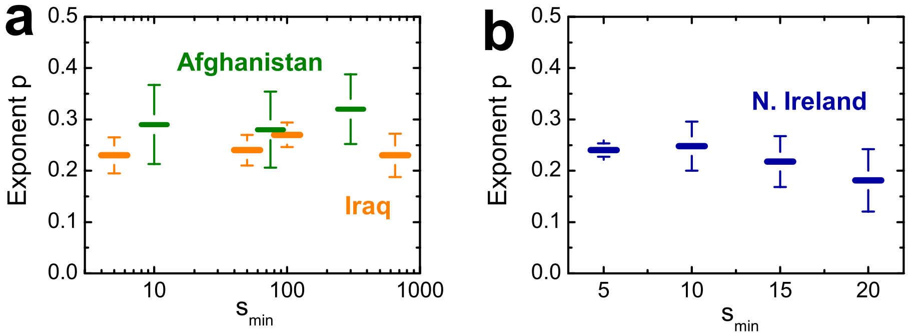

Fig. 3. Robustness of the measured Omori exponent. Power-law exponent p versus $s_{\min}$, with their correspondent 95% confidence intervals. The values of the exponent p remain approximately constant over a given range of $s_{\min}$.

Table 2. Analysis of the time evolution of the averaged cumulative rates, $n_{av}(t-T)$ (aftershocks) and $n'_{av}(|t-T|)$ (foreshocks), for the mainshocks described in Table 1. We list the optimal time of relaxation Δt and the best fitted parameters b and p to equation (2), shown with their correspondent 95% confidence intervals. We also list the coefficient of determination R^2 of the least squares fits. We cannot reject the hypothesis of fit residuals normally distributed (p-values >0.05 are shown). We also show the power-law exponent p_{sh} obtained from shuffled data, with their 95% confidence intervals. According to the Akaike Information Criterion (AIC) test, negative differences δAIC points to equation (2) as the best fit in comparison with an exponential decay. In the analysis of $n'(|t-T|)$, the triggering events 3, 8, 15, 17 were removed due the lack of enough events (foreshocks).

	Afghanistan	Iraq	N. Ireland
Aftershocks			
Δt	115	30	35
b	0.59 ± 0.20	8.05 ± 0.87	0.84 ± 0.03
p	0.29 ± 0.08	0.23 ± 0.03	0.24 ± 0.01
R^2	0.9997	0.9999	0.9999
p-Value	0.14	0.06	0.14
p_{sh}	0.00 ± 0.02	0.000 ± 0.001	0.000 ± 0.003
δAIC	−0.5	−7.7	−20.6
Foreshocks			
Δt	95	30	45
b	0.44 ± 0.10	4.25 ± 0.62	0.49 ± 0.07
p	0.16 ± 0.05	0.04 ± 0.05	0.02 ± 0.04
R^2	0.9999	0.9999	0.9999
p-Value	0.14	0.67	0.14
p_{sh}	0.00 ± 0.02	0.000 ± 0.005	0.00 ± 0.01
δAIC	−2.9	3.6	0.27

zero, $p \simeq 0.16$. For Iraq and N. Ireland data, we have $p \simeq 0$ suggesting absence of activity growth towards the mainshocks.

From a total of 31 mainshocks with enough aftershocks to extract the power-law exponents, the present procedure selected 17 (described in Tab. 1). For other 9 mainshocks, we observed no relaxation in the aftershock rates (power-law exponents close to zero). For the remaining 5 mainshocks, we observed negative exponents suggesting a growth in the aftershock rates. Some of these last findings may be driven by different mechanisms or may be affected by nonstationarities in the activity rate. For this reason, these last 14 mainshocks were not included in the previous analysis.

4 Discussion

By analysing cumulative aftershock rates, we found Omori exponents of the order of 0.2–0.4 for selected triggered events in the Iraq, Afghanistan and N. Ireland conflicts. We also found a precursory power-law growth of activity for the Afghanistan catalog, with an Omori exponent in the same range observed for the aftershock decay rate. For Iraq and N. Ireland catalogs we found no evidence for precursory growth. These findings provide evidence of Omori-like relaxation in violent conflicts, in analogy with previous results in non-violent human activities like financial markets, views of videos and book sales.

Endogenous shocks can be viewed as resulting from the underlying dynamics of the system while exogenous shocks have external sources. In general, the recovery after an endogenous shock is different from the recovery after an exogenous shock, allowing us to distinguish between them. This is important for understanding the relative effects of self-organization versus external impacts. In this sense, it could be instructive to compare our findings on violent conflicts with the predictions of a general model of self-excited processes, such as that discussed in references [18,19,21,29–31].

According to the model mentioned above, the mainshocks can be classified in four categories: exogenous-subcritical, exogenous-critical, endogenous-subcritical and endogenous-critical. The relaxation after an exogenous-subcritical peak of activity exhibits power-law exponents $p = 1+\theta$, with θ in the range 0.3–0.4, giving p close to 1.3–1.4. The relaxation after an exogenous-critical shock (at or close to criticality) is characterized by $p = 1-\theta$, giving p close to 0.6–0.7. For endogenous-subcritical shocks, the system response is basically driven by fluctuations and typical time series of aftershock rates recall a simple stochastic process (or noise). Finally, the relaxation after an endogenous-critical shock (as well as the precursory growth of activity) is a power-law with $p = 1-2\theta$, pointing to p in the range 0.2–0.4. This last range is in remarkable agreement with our findings for the aftershock rates in violent conflicts. For the Afghanistan catalog, the growth of activity towards the peaks of activity was also observed, as the model predicts for endogenous-critical shocks.

Our results are in line with recent findings in violent conflicts, which present similarities with seismic phenomena. For example, robust patterns characterizing violent conflicts have been identified, including power-law size distributions and heterogeneous temporal and spatial behavior [26,32–38]. Some of these findings in collective violence remember general laws governing natural phenomena, like the Gutenberg-Richter law [39,40] and the unified scaling law for earthquakes [41]. Our results suggest that violent conflicts share with earthquakes and other natural and social phenomena a common feature in the dynamics of aftershocks. More specifically, our findings suggest the presence of the Omori's law [6] in collective violence.

A review of the current literature on violent activities including war and crime in general, like the results reported in reference [42] and the reference therein (including [43–50]) and references [51,52], shows that a wide variety of techniques have been applied to quantify violent behaviour, although there are still issues to be solved. We hope the present results, added to the current knowledge on the theme, may help us better understand the mechanisms underlying collective violence.

Author contribution statement

S. Picoli and R.S. Mendes conceived the idea, designed the draft, performed the data analysis. All authors participated in the production of the text.

References

1. J.-P. Bouchaud, J. Phys. I **2**, 1705 (1992)
2. R.V. Chamberlin, Europhys. Lett. **33**, 545 (1996)
3. R. Metzler, E. Barkai, J. Klafter, Phys. Rev. Lett. **82**, 3563 (1999)
4. I.V. Gopich, N. Agmon, Phys. Rev. Lett. **84**, 2730 (2000)
5. L.C. Sampaio et al., Phys. Rev. B **64**, 184440 (2001)
6. F. Omori, J. Coll. Sci. Imp. Univ. Tokyo **7**, 111 (1894)
7. T. Utsu, Y. Ogata, S. Matsu'ura, J. Phys. Earth **43**, 1 (1995)
8. R. Shcherbakov, D.L. Turcotte, J.B. Rundle, Geophys. Res. Lett. **31**, L11613 (2004)
9. S. Zapperi, A. Vespignani, H.E. Stanley, Nature **388**, 658 (1997)
10. J. Baró et al., Phys. Rev. Lett. **110**, 088702 (2013)
11. H.V. Ribeiro et al., Phys. Rev. Lett. **115**, 025503 (2015)
12. A. Johansen, D. Sornette, Physica A **276**, 338 (2000)
13. S. Abe, N. Suzuki, Europhys. Lett. **61**, 852 (2003)
14. F. Lillo, R.N. Mantegna, Phys. Rev. E **68**, 016119 (2003)
15. P. Weber, F. Wang, I. Vodenska-Chitkushev, S. Havlin, H.E. Stanley, Phys. Rev. E **76**, 016109 (2007)
16. A.M. Petersen, F. Wang, S. Havlin, H.E. Stanley, Phys. Rev. E **81**, 066121 (2010)
17. A.M. Petersen, F. Wang, S. Havlin, H.E. Stanley, Phys. Rev. E **82**, 036114 (2010)
18. D. Sornette, F. Deschâtres, T. Gilbert, Y. Ageon, Phys. Rev. Lett. **93**, 228701 (2004)
19. F. Deschâtres, D. Sornette, Phys. Rev. E **72**, 016112 (2005)
20. R. Lambiotte, M. Ausloos, Physica A **362**, 485 (2006)
21. R. Crane, D. Sornette, Proc. Natl. Acad. Sci. **105**, 15649 (2008)
22. M.P. Ciamarra, A. Coniglio, L. de Arcangelis, Europhys. Lett. **84**, 28004 (2008)
23. P. Klimek, W. Bayer, S. Thurner, Physica A **390**, 3870 (2011)
24. D. Sornette, *Extreme Events in Nature and Society* (Springer, Heidelberg), pp. 95–119.
25. P. Meier, D. Sornette, R. Woodard, *Paper Presented at the 49th International Studies Association Annual Convention* (San Francisco, CA, 2008)
26. J.C. Bohorquez, S. Gourley, A.R. Dixon, M. Spagat, N.F. Johnson, Nature **462**, 911 (2009)
27. S. Carran, A. Ravindar, S.Y. Lau, J. Bohannon, https://www.sciencemag.org/content/331/6022/1256/suppl/DC1 (2011)
28. M. Sutton, http://cain.ulst.ac.uk/sutton
29. A. Helmstetter, D. Sornette, J. Geophys. Res. **107**, 2237 (2002)
30. D. Sornette, A. Helmstetter, Physica A **318**, 577 (2003)
31. A. Saichev, D. Sornette, Eur. Phys. J. B **75**, 343 (2010)
32. L.F. Richardson, J. Am. Stat. Assoc. **43**, 523 (1948)
33. L.-E. Cederman, Am. Polit. Sci. Rev. **97**, 135 (2003)
34. A. Clauset, M. Young, K.S. Gleditsch, J. Confl. Resolut. **51**, 58 (2007)
35. N. Johnson et al., Science **333**, 81 (2011)
36. A. Clauset, K.S. Gleditsch, PLoS ONE **7**, e48633 (2012)
37. N. Johnson et al., Sci. Rep. **3**, 3463 (2013)
38. S. Picoli, M. Castillo-Mussot, H.V. Ribeiro, E.K. Lenzi, R.S. Mendes, Sci. Rep. **4**, 4773 (2014)
39. B. Gutenberg, C. Richter, *Seismicity of the Earth* (Princeton University, Princeton, 1949)
40. D.L. Turcotte, *Fractals and Chaos in Geology and Geophysics* (Cambridge University Press, Cambridge, 1997)
41. P. Bak, K. Christensen, L. Danon, T. Scanlon, Phys. Rev. Lett. **88**, 178501 (2002)
42. M.R. D'Orsogna, M. Perc, Phys. Life Rev. **12**, 1 (2015)
43. L.G.A. Alves, H.V. Ribeiro, E.K. Lenzi, R.S. Mendes, PLoS ONE **8**, e69580 (2013)
44. L.G.A. Alves, H.V. Ribeiro, R.S. Mendes, Physica A **392**, 2672 (2013)
45. E. Lewis, G. Mohler, P.J. Brantingham, A.L. Bertozzi, Secur. J. **25**, 244 (2012)
46. G.O. Mohler, M.B. Short, P.J. Brantingham, F.P. Schoenberg, G.E. Tita, J. Am. Stat. Assoc. **106**, 100 (2011)
47. M.B. Short, A.L. Bertozzi, P.J. Brantingham, SIAM J. Appl. Dyn. Syst. **9**, 462 (2010)
48. M.B. Short, P.J. Brantingham, A.L. Bertozzi, G.E. Tita, Proc. Natl. Acad. Sci. USA **107**, 3961 (2010)
49. M.B. Short et al., Math. Models Methods Appl. Sci. **18**, 1249 (2008)
50. M. Townsley, R. Homel, J. Chaseling, Aust. NZ J. Criminol. **33**, 37 (2000)
51. L.G.A. Alves, E.K. Lenzi, R.S. Mendes, H.V. Ribeiro, Europhys. Lett. **111**, 18002 (2015)
52. Q.S. Hanley, D. Lewis, H.V. Ribeiro, PLoS ONE **11**, e0149546 (2016)